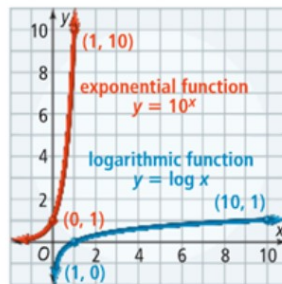


Logarithmic Functions



Objectives To write and evaluate logarithmic expressions



Take note

Key Concept Logarithm

A **logarithm** base b of a positive number x satisfies the following definition.

$$\text{For } b > 0, b \neq 1, \log_b x = y \text{ if and only if } b^y = x.$$

You can read $\log_b x$ as “log base b of x .” In other words, the logarithm y is the exponent to which b must be raised to get x .

A **logarithmic function** is the inverse of an exponential function.

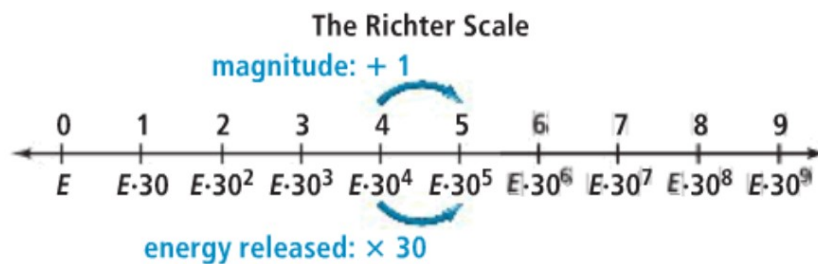
A **common logarithm** is a logarithm with base 10. You can write a common logarithm $\log_{10} x$ simply as $\log x$, without showing the 10.

The function $y = e^x$ has an inverse, the **natural logarithmic function**, $y = \log_e x$, or $y = \ln x$.

Logarithmic Functions

A **common logarithm** is a logarithm with base 10. You can write a common logarithm $\log_{10}x$ simply as $\log x$, without showing the 10.

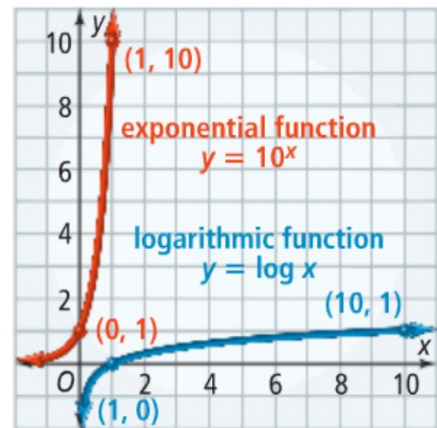
Many measurements of physical phenomena have such a wide range of values that the reported measurements are logarithms (exponents) of the values, not the values themselves. When you use the logarithm of a quantity instead of the quantity, you are using a **logarithmic scale**. The Richter scale is a logarithmic scale. It gives logarithmic measurements of earthquake magnitude.



Logarithmic Functions

A **logarithmic function** is the inverse of an exponential function. The graph shows $y = 10^x$ and its inverse $y = \log x$. Note that $(0, 1)$ and $(1, 10)$ are on the graph of $y = 10^x$, and that $(1, 0)$ and $(10, 1)$ are on the graph of $y = \log x$.

Recall that the graphs of inverse functions are reflections of each other across the line $y = x$. You can graph $y = \log_b x$ as the inverse of $y = b^x$.



Exponential Equation

$$b^x = a$$

Logarithmic Equation

$$\log_b a = x$$

$$b > 0, b \neq 1$$

Logarithmic Functions

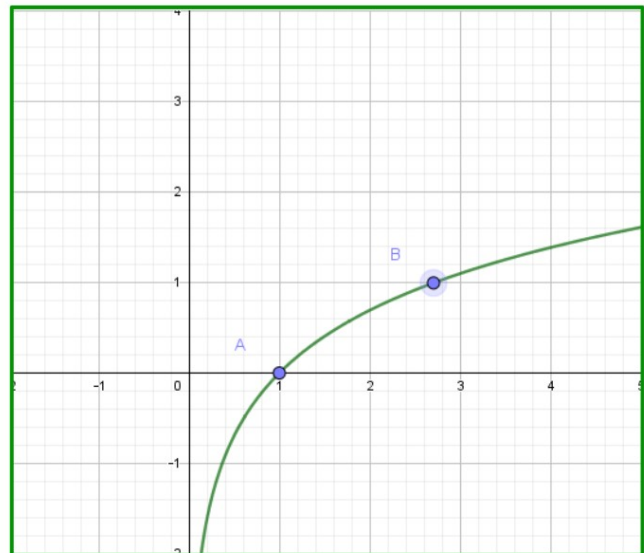
The parent function $y = \log_{10}(x)$

Domain: $\{x|x > 0\}$

Range: $\{y|-\infty < y < +\infty\}$

The asymptote : ***y-axis***

The reference points : ***(1, 0)***
(e, 1)



Logarithmic Functions

take note

Concept Summary Families of Logarithmic Functions

Parent functions:

$$y = \log_b x, b > 0, b \neq 1$$

Stretch ($|a| > 1$)

Compression (Shrink) ($0 < |a| < 1$)

Reflection ($a < 0$) in x -axis

}

$$y = a \log_b x$$

Translations (horizontal by h ; vertical by k)

$$y = \log_b (x - h) + k$$

All transformations together

$$y = a \log_b (x - h) + k$$

Logarithmic Functions

If $x = b^y$ then $\log_b x = y$

What is the Logarithmic form of the following?

A $100 = 10^2$

Use the definition of logarithm.

If $x = b^y$ then $\log_b x = y$

If $100 = 10^2$ then $\log_{10} 100 = 2$

Logarithmic Functions

If $x = b^y$ then $\log_b x = y$

What is the Logarithmic form of the following?

B $81 = 3^4$

Use the definition of logarithm.

If $x = b^y$ then $\log_b x = y$

If $81 = 3^4$ then $\log_3 81 = 4$

Logarithmic Functions

If $x = b^y$ then $\log_b x = y$



Got It?

1. What is the logarithmic form of each equation?

a. $36 = 6^2$

b. $\frac{8}{27} = \left(\frac{2}{3}\right)^3$

c. $1 = 3^0$

Logarithmic Functions



Problem 2 Evaluating a Logarithm

What is the value of $\log_8 32$?

$\log_8 32 = x$ Write a logarithmic equation.

$32 = 8^x$ Use the definition of a logarithm to write an exponential equation.

$2^5 = (2^3)^x$ Write each side using base 2.

$2^5 = 2^{3x}$ Power Property of Exponents

$5 = 3x$ Since the bases are the same, the exponents must be equal.

$\frac{5}{3} = x$ Solve for x .

Logarithmic Functions



Got It? 2. What is the value of each logarithm?

a. $\log_5 125$

b. $\log_4 32$

c. $\log_{64} \frac{1}{32}$

Logarithmic Functions



Problem 3 Using a Logarithmic Scale

In December 2004, an earthquake with magnitude 9.3 on the Richter scale hit off the northwest coast of Sumatra. The diagram shows the magnitude of an earthquake that hit Sumatra in March 2005. The formula $\log \frac{I_1}{I_2} = M_1 - M_2$ compares the intensity levels of earthquakes where I is the intensity level determined by a seismograph, and M is the magnitude on a Richter scale. How many times more intense was the December earthquake than the March earthquake?

$$\log \frac{I_1}{I_2} = M_1 - M_2 \quad \text{Use the formula.}$$

$$\log \frac{I_1}{I_2} = 9.3 - 8.7 \quad \text{Then Plug}$$

$$\log \frac{I_1}{I_2} = 0.6 \quad \text{and Chug}$$

$$\frac{I_1}{I_2} = 10^{0.6}$$



Logarithmic Functions

The acidity level, or pH, of a liquid is given by the formula $\text{pH} = \log \frac{1}{[\text{H}^+]}$ where $[\text{H}^+]$ is the concentration (in moles per liter) of hydrogen ions in the liquid. In a typical chlorinated swimming pool, the concentration of hydrogen ions ranges from 1.58×10^{-8} moles per liter to 6.31×10^{-8} moles per liter. What is the range of the pH for a typical swimming pool?



Using the pH formula, substitute the given values of $[\text{H}^+]$.

$$\text{pH} = \log\left(\frac{1}{6.31 \times 10^{-8}}\right) \quad \text{and} \quad \text{pH} = \log\left(\frac{1}{1.58 \times 10^{-8}}\right)$$
$$\approx \log 15,800,000 \qquad \qquad \approx \log 63,300,000$$

So, the pH of a swimming pool ranges from 7.2 to 7.8.

Logarithmic Functions

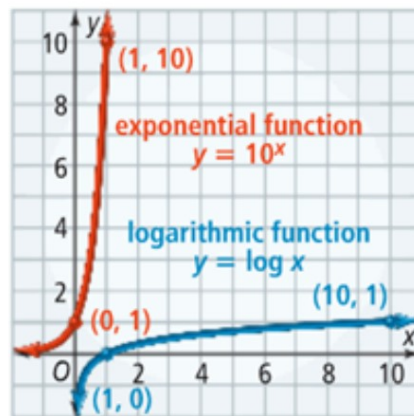
ANY QUESTIONS

Logarithmic Functions

Objectives To write and evaluate logarithmic expressions
To graph logarithmic functions

Classwork

Worksheet 15.1



If $x = b^y$ then $\log_b x = y$

Essential Understanding The exponential function $y = b^x$ is one-to-one, so its inverse $x = b^y$ is a function. To express “ y as a function of x ” for the inverse, write $y = \log_b x$.

Common Log 3

Natural Log LN is log to the base e

Log_e i e π 1 0

$$\text{Log}_7 12 = \log(12)/\log(7)$$

$$\text{Log}_3 81 = \log(81)/\log(3)$$